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by Fred R. DeJarnette and Ruby M. Davis

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A SIMPLIFIED METHOD FOR CALCULATING LAMINAR HEAT TRANSFER OVER BODIES AT AN ANGLE OF ATTACK

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SUMMARY

A simplified method is developed for calculating the ratio of the local to stagnation-point heat-transfer rate for bodies at an angle of attack with only the free-stream Mach number and the ratio of specific heats required as inputs. The viscous problem is simplified by using the axisymmetric analog for three-dimensional boundary layers (small cross flow in boundary layer) in conjunction with Lees' laminar heating rate for axisymmetric bodies. An approximate technique for determining the geometry of the inviscid surface streamline is presented. In this technique the direction of a streamline at a given point on the body is taken as the direction of the resultant of the free-stream velocity vector minus its normal component. For determining the inviscid surface properties, the modified Newtonian pressure distribution is used with isentropic flow along the surface.

The heat transfer over spherically blunted cones with 15° and 30° semiapex angles at angles of attack of 0°, 10°, and 20° and a free-stream Mach number of 10.6 was computed by the present method. The values obtained agreed well with experimental data.

INTRODUCTION

Theoretical methods for predicting the aerodynamic heat transfer to symmetric bodies at an angle of attack and to lifting bodies are required for the proper design of reentry heat-protection systems. A review of the literature (see, for instance, ref. 1) shows that previous methods are restricted to small angles of attack or to yawed infinite cylinders, or that they involve complicated numerical techniques to solve the governing flow equations. This paper presents a relatively simple method for computing the laminar convective heating rates over basic hypersonic configurations at an angle of attack and with a highly cooled surface. Radiative heat transfer is not considered here, nor is the effect of ablation gases on the convective heating rates.

The complexity of the three-dimensional differential equations governing the inviscid as well as the viscous flow field over a body at an angle of attack makes the use of simplifying approximations desirable so that tractable solutions may be obtained. A substantial simplification to the viscous flow field equations is achieved through the axisymmetric analog for three-dimensional boundary layers (refs. 2 and 3). This axisymmetric analog transforms the general three-dimensional boundary-layer equations into the same form as those for an axisymmetric body at an angle of attack of 0° when the coordinate directions are taken along and perpendicular to the inviscid body streamlines, and when the cross flow (i.e., the component of boundary-layer flow normal to an inviscid streamline and along the body surface) is assumed to be small. Therefore, if the pressure distribution and streamline geometry are known, the heat transfer may be calculated along a streamline by any method applicable to a body of revolution at an angle of attack of 0°; such as the method of Lester Lees (ref. 4).

The major difficulty in applying the axisymmetric analog to angle-of-attack problems is that the inviscid solution on the body surface (surface pressures, velocity, and streamline geometry) is required. Ordinarily, the inviscid solution would have to be obtained by some numerical technique, which is a major undertaking in itself for bodies at an angle of attack. Therefore, some approximations in the inviscid solution are, in many cases, necessary.

In the present method, the direction of an inviscid streamline at a given point on the body is obtained by assuming that the free-stream velocity vector loses its normal component upon striking the body. Thus, the direction of a streamline is taken as the direction of the resultant of the free-stream velocity vector minus its normal component at a given body point. Once the expression for the streamline direction is obtained, a first-order ordinary differential equation for the geometry of the streamline may be written. Then, another ordinary differential equation for the scale factor (corresponding to the body radius in the axisymmetric analog and often referred to as the streamline divergence function) is obtained. Both these differential equations may be readily integrated in closed form for some simple body shapes; and, for more complicated body shapes, they may be integrated by a simple numerical scheme such as the Runge-Kutta method.

The magnitude of the flow velocity along a streamline is determined directly from the pressure distribution under the assumption that the inviscid flow on the surface is isentropic. In the present paper, modified Newtonian pressure distributions are assumed for convenience.

The accuracy of the present method was partially assessed by computing the ratio of local to stagnation-point heat-transfer rate on spherically blunted 15° and 30° cones at a free-stream Mach number of 10.6, a ratio of free-stream specific heats of 1.4, and angles of attack of 0°, 10°, and 20°. The results are shown to compare well with experimental data.

SYMBOLS

A	function defined by equation (10)
$\hat{e}_n, \hat{e}_s$	unit vectors normal (outer) to the body surface and along inviscid surface streamline, respectively
$\hat{e}_\beta$	unit vector normal to $\hat{e}_n$ and $\hat{e}_s$, defined by equation (12)
$\hat{e}_r, \hat{e}_\phi$	unit vectors in r and ϕ directions, respectively
G	function defined by equation (3)
H	scale factor for ξ coordinate
h	scale factor for β coordinate
$\hat{i}, \hat{j}, \hat{k}$	unit vectors in x, y, and z directions, respectively
k	slope of cone surface
l	metric defined by equation (5)
M_∞	free-stream Mach number
n	coordinate normal to body surface
p	local static pressure
p_o	stagnation pressure behind normal shock wave
$\dot{q}_w$	local heat-transfer rate
R	nose radius
r	body radius
r_b	body base radius

s	distance along a streamline, measured from the stagnation point
u_e	inviscid velocity along body-surface streamline
V_∞	undisturbed free-stream velocity
$\vec{V}_\infty$	undisturbed free-stream velocity vector
x,y,z	Cartesian coordinates (fig. 1)
α	angle of attack
β	coordinate measured along body surface, perpendicular to inviscid streamlines
γ_∞	ratio of specific heats in free stream
$\bar{\gamma}$	mean ratio of specific heats behind bow shock wave
θ	inclination of body surface with respect to body axis
ξ	coordinate measured along a streamline, $H\,d\xi = ds$
ϕ	angular coordinate (fig. 1)
ψ	angle between $\vec{V}_\infty$ and $-\hat{e}_n$, defined by equation (20)

Subscripts:

i	sphere-cone interface
o	stagnation point
x	indicates that derivative is taken with x held constant
β	indicates that derivative is taken with β held constant

A prime indicates a derivative with respect to x.

ANALYSIS

Problem Description

For axisymmetric blunt bodies at an angle of attack of 0° and with a highly cooled surface, the ratio of the local heat-transfer rate $\dot{q}_w$ to the stagnation-point heat-transfer rate $\dot{q}_{w,o}$ given in the notation of the present report, was derived in reference 4:

$$\frac{\dot{q}_w}{\dot{q}_{w,o}} = \frac{(p/p_o)(u_e/V_\infty) r R^{1/2}}{\left[\int_0^s (p/p_o)(u_e/V_\infty) r^2 \, ds\right]^{1/2} 2G} \tag{1}$$

The modified Newtonian pressure distribution combined with the assumption of isentropic flow along the body surface yields the following results (ref. 4):

$$\frac{p}{p_o} \approx \left(1 - \frac{1}{\gamma_\infty M_\infty{}^2}\right)\cos^2\psi + \frac{1}{\gamma_\infty M_\infty{}^2} \tag{2}$$

$$G = \left[\frac{1}{V_\infty}\left(\frac{du_e}{d\theta}\right)_o\right]^{1/2} \approx \left[\frac{\bar{\gamma} - 1}{\bar{\gamma}}\left(1 + \frac{2}{(\gamma_\infty - 1)M_\infty{}^2}\right)\left(1 - \frac{1}{\gamma_\infty M_\infty{}^2}\right)\right]^{1/4} \tag{3}$$

$$\frac{u_e}{V_\infty} = \left\{\left[1 + \frac{2}{(\gamma_\infty - 1)M_\infty{}^2}\right]\left[1 - \left(\frac{p}{p_o}\right)^{(\bar{\gamma} - 1)/\bar{\gamma}}\right]\right\}^{1/2} \tag{4}$$

According to the axisymmetric analog (refs. 2 and 3), equations (1) to (4) are also applicable to any inviscid surface streamline on an asymmetric body or an axisymmetric body at an angle of attack if s is the distance measured along the streamline and r is replaced by the scale factor h corresponding to the coordinate β measured along the body surface and perpendicular to the streamline. The assumption of zero cross flow, which is implied in the axisymmetric analog, was shown in reference 5 to be applicable to highly cooled blunt bodies in hypersonic flow.

The present analysis will consider a spherically blunted axisymmetric body at an angle of attack. The basic method presented here is applicable, however, to any three-dimensional body at an angle of attack. The coordinates ξ, β, and n (when n is measured from the surface along a straight line normal to the surface) form an orthogonal coordinate system whose metric is

$$dl^2 = H^2 d\xi^2 + h^2 d\beta^2 + dn^2 \tag{5}$$

where $H \, d\xi = ds$.

On the body surface $n = 0$, $H = H(x,\phi)$, and $h = h(x,\phi)$, where x is the distance along the body axis and ϕ is the circumferential position. (See fig. 1.) The coordinate β is constant along a given inviscid surface streamline.

If the modified Newtonian pressure distribution (eq. (2)) is accepted as being sufficiently accurate, then the heating-rate ratio for the body at an angle of attack may be computed from equations (1) to (4) along any inviscid surface streamline, provided the streamline geometry and scale factor h are known (H is not required).

Streamline Geometry

In order to determine the streamline geometry and scale factor h, it is assumed here that the direction of an inviscid surface streamline may be taken as the direction of the resultant of the free-stream velocity vector minus its normal component at a given point on the body surface. With $\hat{e}_s$ defined as a unit vector in the direction of a streamline, this assumption gives

$$\hat{e}_s = \frac{\vec{V}_\infty - \left(\vec{V}_\infty \cdot \hat{e}_n\right)\hat{e}_n}{\left|\vec{V}_\infty - \left(\vec{V}_\infty \cdot \hat{e}_n\right)\hat{e}_n\right|} \tag{6}$$

where $\hat{e}_n$ is a unit vector normal (outer) to the surface, which is given by

$$\hat{e}_n = \frac{-\hat{i}r' + \hat{j}\cos\phi + \hat{k}\sin\phi}{\left(1 + r'^2\right)^{1/2}} \tag{7}$$

The quantities $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x, y, and z directions, respectively (fig. 1). As shown in figure 1, $\vec{V}_\infty$ lies in the xy-plane; hence

$$\vec{V}_\infty = V_\infty\left(\hat{i}\cos\alpha + \hat{j}\sin\alpha\right) \tag{8}$$

where α is the angle of attack. Equations (7) and (8) may be utilized in equation (6) to obtain

$$\hat{e}_s = \frac{\left(\hat{i} + r'\hat{e}_r\right)\left(\cos\alpha + r'\sin\alpha\cos\phi\right) - \hat{e}_\phi\left(1 + r'^2\right)\sin\alpha\sin\phi}{\left(1 + r'^2\right)^{1/2}A} \tag{9}$$

where

$$A = \left[\sin^2\phi + \left(r'\sin\alpha + \cos\alpha\cos\phi\right)^2\right]^{1/2} \tag{10}$$

and $\hat{e}_r$ and $\hat{e}_\phi$ are unit vectors in the directions of r and ϕ, respectively. (See fig. 1.)

The differential equation that defines the streamline geometry may now be formed from equation (9) as

$$r\left(\frac{\partial \phi}{\partial x}\right)_\beta = \frac{\hat{e}_s \cdot \hat{e}_\phi}{\hat{e}_s \cdot \hat{i}} = \frac{-(1 + r'^2)\sin \phi \sin \alpha}{\cos \alpha + r' \cos \phi \sin \alpha} \tag{11}$$

Although the left side of this equation is written as a partial derivative, β is constant along a streamline and therefore it may be replaced by an ordinary derivative for the integration of a given streamline. Thus, for any chosen streamline emanating from the stagnation point, equation (11) may be integrated to determine the geometry of this streamline (that is, $\phi = \phi(x)$) as it wraps around the body. Note that the modified Newtonian pressure distribution locates the stagnation point at $r_o' = \cot \alpha$, $\phi = \pi$ radians.

Scale Factor

The next task is to determine a relation for the scale factor h along a streamline. Let $\hat{e}_\beta$ be a unit vector in the direction of β; thus

$$\hat{e}_\beta = \hat{e}_s \times \hat{e}_n$$

$$\hat{e}_\beta = \frac{\left(\hat{i} + \hat{e}_r r'\right)\sin \alpha \sin \phi + \hat{e}_\phi(r' \sin \alpha \cos \phi + \cos \alpha)}{A} \tag{12}$$

Now since the element of arc length in the β direction on the surface is $h\, d\beta$, then

$$\frac{h}{r}\left(\frac{\partial \beta}{\partial \phi}\right)_x = \hat{e}_\beta \cdot \hat{e}_\phi = \frac{r' \sin \alpha \cos \phi + \cos \alpha}{A} \tag{13}$$

Therefore, along the surface

$$h = \frac{r(r' \sin \alpha \cos \phi + \cos \alpha)}{A}\left(\frac{\partial \phi}{\partial \beta}\right)_x \tag{14}$$

where $\phi = \phi(x,\beta)$ defines the circumferential location of the inviscid surface streamlines, and again β is constant along a given streamline.

Since $\phi = \phi(x,\beta)$ is analytic,

$$\frac{\partial^2 \phi}{\partial x\, \partial \beta} = \frac{\partial^2 \phi}{\partial \beta\, \partial x} \tag{15}$$

and since $\left(\frac{\partial}{\partial \beta}\right)_x = \left(\frac{\partial}{\partial \phi}\right)_x \left(\frac{\partial \phi}{\partial \beta}\right)_x$,

$$\frac{\partial^2 \phi}{\partial \beta\, \partial x} = \left[\frac{\partial}{\partial \phi}\left(\frac{\partial \phi}{\partial x}\right)_\beta\right]_x \left(\frac{\partial \phi}{\partial \beta}\right)_x \tag{16}$$

Then equation (15) becomes

$$\left[\frac{\partial}{\partial x}\ln\left(\frac{\partial\phi}{\partial\beta}\right)_x\right]_\beta = \left[\frac{\partial}{\partial\phi}\left(\frac{\partial\phi}{\partial x}\right)_\beta\right]_x \tag{17}$$

Taking the partial derivative of equation (11) with respect to ϕ and holding x constant yields

$$\left[\frac{\partial}{\partial\phi}\left(\frac{\partial\phi}{\partial x}\right)_\beta\right]_x = \frac{-(1 + r'^2)(r' \tan\alpha + \cos\phi)\tan\alpha}{r(1 + r' \cos\phi \tan\alpha)^2}$$

With this substitution, equation (17) finally becomes

$$\left[\frac{\partial}{\partial x}\ln\left(\frac{\partial\phi}{\partial\beta}\right)_x\right]_\beta = \frac{-(1 + r'^2)(r' \tan\alpha + \cos\phi)\tan\alpha}{r(1 + r' \cos\phi \tan\alpha)^2} \tag{18}$$

For the integration of equation (18) along a given streamline, the left side of this equation may be written as an ordinary derivative:

$$\frac{d}{dx}\left[\ln\left(\frac{\partial\phi}{\partial\beta}\right)_x\right] = \frac{-(1 + r'^2)(r' \tan\alpha + \cos\phi)\tan\alpha}{r(1 + r' \cos\phi \tan\alpha)^2} \tag{19}$$

The solution of this equation allows h to be computed from equation (14).

Equations (2) and (4) may also be used to compute the pressure ratio and velocity ratio along any inviscid surface streamline for the body at an angle of attack. For equation (2), $\cos\psi$ may be evaluated from equations (7) and (8) as

$$\cos\psi = -\frac{\vec{V}_\infty \cdot \hat{e}_n}{V_\infty} = \frac{r' \cos\alpha - \cos\phi \sin\alpha}{(1 + r'^2)^{1/2}} \tag{20}$$

However, the surface region on the leeward side of the body at an angle of attack where $\cos\psi \leqq 0$ will be in the "aerodynamic shadow," and the modified Newtonian theory does not predict the pressures in this region. In the present method the surface pressures in the shadowed region are assumed to be free-stream static pressure, but the streamline geometry and the scale factor h are still determined from equations (11), (14), and (19) in this region.

Application to Spherically Blunted Cone at Angle of Attack

The determination of the streamline geometry and the scale factor h is very simple for spherically capped bodies at an angle of attack when the surface sonic line (inviscid) lies entirely on the spherical surface. In this case the streamlines on a portion of the spherical cap follow spherical meridians about an axis through the center of the sphere and parallel to $\vec{V}_\infty$. From geometrical considerations, the coordinate β on the spherical cap is the angular position of a streamline measured in the plane tangent to the spherical cap at the stagnation point, with $\beta = 0$ for the top meridian and $\beta = \pi$ radians for the bottom meridian. (See fig. 2.) Also, while on the spherical cap, the scale factor h is simply the distance from the point in question on the sphere to the axis through the center of the sphere and parallel to $\vec{V}_\infty$. Therefore, for the spherical cap,

$$\frac{h}{R} = \sin \psi = \left[\frac{r'^2 + 1 - (r' \cos \alpha - \cos \phi \sin \alpha)^2}{r'^2 + 1}\right]^{1/2} \tag{21}$$

and from the geometry of the streamlines, it may be verified that at the sphere-cone interface β and ϕ_i are related by the expression

$$h_i \sin \beta = r_i \sin \phi_i$$

Hence it follows that

$$\cos \beta = \frac{\cos \phi_i + k \tan \alpha}{\left[(1 + k^2)\sin^2\phi_i \tan^2\alpha + \left(1 + k \cos \phi_i \tan \alpha\right)^2\right]^{1/2}} \tag{22}$$

where i refers to values at the sphere-cone interface and k is the slope of the cone surface:

$$k = \tan \theta = r' \tag{23}$$

For the conical afterbody, equation (11) may be integrated to yield the streamline geometry as

$$\frac{r}{r_i} = \left(\frac{\tan \frac{\phi_i}{2}}{\tan \frac{\phi}{2}}\right)^{\frac{k}{(1+k^2)\tan \alpha}} \left(\frac{\sin \phi_i}{\sin \phi}\right)^{\frac{k^2}{1+k^2}} \tag{24}$$

To determine the scale factor h for the afterbody, it is noted that

$$\left(\frac{\partial \phi}{\partial \beta}\right)_x = \left(\frac{\partial \phi}{\partial \phi_i}\right)_x \left(\frac{\partial \phi_i}{\partial \beta}\right)_x \tag{25}$$

and taking the partial derivative of equation (24) with respect to ϕ_i (holding x constant) yields

$$\left(\frac{\partial \phi}{\partial \phi_i}\right)_x = \left(\frac{1 + k \tan \alpha \cos \phi_i}{1 + k \tan \alpha \cos \phi}\right)\frac{\sin \phi}{\sin \phi_i} \tag{26}$$

Now equation (22) is differentiated with respect to β (holding x constant) to get

$$\left(\frac{\partial\phi_i}{\partial\beta}\right)_x = \frac{\sin^2\phi_i + \left(\cos\phi_i \cos\alpha + k\sin\alpha\right)^2}{\cos\alpha + k\sin\alpha\cos\phi_i} \tag{27}$$

Finally, for the conical afterbody, equations (24) to (27) are substituted into equation (14) to obtain

$$\frac{h}{R} = \left(\frac{\tan\frac{\phi_i}{2}}{\tan\frac{\phi}{2}}\right)^{\frac{k}{(1+k^2)\tan\alpha}} \left(\frac{\sin\phi}{\sin\phi_i}\right)^{\frac{1}{1+k^2}} \frac{\sin^2\phi_i + \left(\cos\phi_i\cos\alpha + k\sin\alpha\right)^2}{\left[\sin^2\phi + (\cos\phi\cos\alpha + k\sin\alpha)^2\right]^{1/2}(1+k^2)^{1/2}} \tag{28}$$

The reduction of equation (28) for the limiting condition of $\alpha = 0$ and for the case of $\phi = 0, \pi$ with α finite is given in the appendix.

Equations (24) and (28) may now be used in equations (1) to (4), with r replaced by h and s measured along an inviscid surface streamline, to determine the heating-rate ratio along any streamline. The integral in the denominator of equation (1) can be evaluated by any simple technique, such as Simpson's rule.

RESULTS

The method presented here was used to compute the nondimensional heating rate over spherically blunted cones with 15° and 30° semiapex angles. Angles of attack of 0°, 10°, and 20°, a Mach number of 10.6, and a ratio of specific heats of 1.4 were considered. Calculated variations have been compared with the experimental data given in reference 6.

Figures 3 and 4 give the longitudinal variation of the nondimensional heating rate along the bottom, top, and side $\left(\phi = \frac{\pi}{2} \text{ radians}\right)$ at angles of attack of 0°, 10°, and 20°. It is clear from these figures that theoretical and experimental data agree well along the windward and side meridians, but along the top meridian (leeward side) the agreement is poor. Also, near the spherical cap theoretical values generally exceed experimental data. This would seem to indicate that the spherical-cap heating-rate distributions are not predicted accurately near the sphere-cone interface.

Variations around the bodies at specific longitudinal locations are given in figures 5 and 6. Previous observations made concerning the longitudinal variations are again valid; that is, the agreement between experimental and theoretical data is worst on the leeward side and near the nose cap.

It should be noted that when the angle of attack is large enough to cause any portion of the sonic line to move off the spherical cap, the stagnation point will move a short distance from the position $r_o' = \cot\alpha$, $\phi = \pi$ radians (see ref. 7). In addition, the axial symmetry on a portion of the spherical cap, about an axis through the center of the sphere and parallel to $\vec{V}_\infty$, is destroyed. The present method does not account for this phenomenon, and the spherically blunted 30° cone at $\alpha = 20^\circ$ is a marginal case.

CONCLUDING REMARKS

A simplified method for computing the laminar heating rates over bodies at an angle of attack is developed and the results are shown to agree well with experimental results for the case of a spherically blunted cone. Although the applications presented here are blunted cones, the basic method is applicable to any asymmetric or axisymmetric body at an angle of attack. The relatively small amount of numerical computations required makes the method particularly attractive for engineering applications.

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APPENDIX

LIMITING FORM OF SCALE FACTOR h FOR $\alpha = 0$ AND FOR $\phi = 0, \pi$ WITH $\alpha > 0$

Equation (24) may be used to rewrite equation (28) in the following form:

$$\frac{h}{R} = \frac{r}{r_i}\frac{\sin \phi}{\sin \phi_i}\frac{\sin^2\phi_i + (\cos \phi_i \cos \alpha + k \sin \alpha)^2}{\left[\sin^2\phi + (\cos \phi \cos \alpha + k \sin \alpha)^2\right]^{1/2}(1 + k^2)^{1/2}} \tag{A1}$$

Then if equation (24) is rearranged to give

$$\left(\frac{r}{r_i}\right)^{\frac{(1+k^2)\tan \alpha}{k}} = \frac{\tan \frac{\phi_i}{2}}{\tan \frac{\phi}{2}}\left(\frac{\sin \phi_i}{\sin \phi}\right)^{k \tan \alpha} \tag{A2}$$

it is clear that in the limit as $\alpha \to 0$,

$$1 = \frac{\tan \frac{\phi_i}{2}}{\tan \frac{\phi}{2}}$$

or

$$\phi_i = \phi$$

With this identity substituted into equation (A1), there results

$$\left(\frac{h}{R}\right)_{\alpha=0} = \frac{r}{r_i(1 + k^2)^{1/2}} \tag{A3}$$

But $r_i = R(1 + k^2)^{-1/2}$ and the correct limit for h as $\alpha \to 0$ is obviously obtained:

$$h_{\alpha=0} = r$$

The next problem is to find the limit of h as $\phi \to 0$ and as $\phi \to \pi$ radians (for $\alpha > 0$). From equation (A1), it is found for the first limit that

$$\lim_{\substack{\phi \to 0 \\ \phi_i \to 0}}\left(\frac{h}{R}\right) = \frac{r}{r_i}\frac{(1 + k \tan \alpha)\cos \alpha}{(1 + k^2)^{1/2}}\lim_{\substack{\phi \to 0 \\ \phi_i \to 0}}\left(\frac{\sin \phi}{\sin \phi_i}\right) \tag{A4}$$

APPENDIX

In order to evaluate $\lim\limits_{\substack{\phi\to 0\\ \phi_i\to 0}}\left(\frac{\sin\phi}{\sin\phi_i}\right)$, the trigonometric identity $\tan\frac{\phi}{2} = \frac{\sin\phi}{1+\cos\phi}$ is substituted into equation (24) to get

$$\frac{r}{r_i} = \left(\frac{1+\cos\phi}{1+\cos\phi_i}\right)^{\frac{k}{(1+k^2)\tan\alpha}}\left(\frac{\sin\phi_i}{\sin\phi}\right)^{\frac{k+k^2\tan\alpha}{(1+k^2)\tan\alpha}} \tag{A5}$$

Then from equation (A5),

$$\lim_{\substack{\phi\to 0\\ \phi_i\to 0}}\left(\frac{\sin\phi}{\sin\phi_i}\right) = \left(\frac{r}{r_i}\right)^{-\frac{(1+k^2)\tan\alpha}{k+k^2\tan\alpha}}$$

Now substitute this relationship into equation (A4) to get

$$\lim_{\substack{\phi\to 0\\ \phi_i\to 0}}\left(\frac{h}{R}\right) = \left(\frac{r}{r_i}\right)^{\frac{k-\tan\alpha}{k+k^2\tan\alpha}}\frac{(1+k\tan\alpha)\cos\alpha}{(1+k^2)^{1/2}} \tag{A6}$$

Next consider $\lim\limits_{\substack{\phi\to\pi\\ \phi_i\to\pi}}\left(\frac{h}{R}\right)$. From equation (A1),

$$\lim_{\substack{\phi\to\pi\\ \phi_i\to\pi}}\left(\frac{h}{R}\right) = \frac{r}{r_i}\,\frac{(1-k\tan\alpha)\cos\alpha}{(1+k^2)^{1/2}}\lim_{\substack{\phi\to\pi\\ \phi_i\to\pi}}\left(\frac{\sin\phi}{\sin\phi_i}\right) \tag{A7}$$

Note that since the stagnation point must lie on the spherical cap and since $r_o' = \cot\alpha$ at the stagnation point, $k\tan\alpha < 1$ everywhere on the conical afterbody.

Now the trigonometric identity $\tan\frac{\phi}{2} = \frac{1-\cos\phi}{\sin\phi}$ is substituted into equation (24) to get

$$\frac{r}{r_i} = \left(\frac{1-\cos\phi_i}{1-\cos\phi}\right)^{\frac{k}{(1+k^2)\tan\alpha}}\left(\frac{\sin\phi_i}{\sin\phi}\right)^{\frac{k^2\tan\alpha-k}{(1+k^2)\tan\alpha}}$$

Then in the limit this equation goes to

$$\lim_{\substack{\phi \to \pi \\ \phi_i \to \pi}} \left(\frac{\sin \phi}{\sin \phi_i}\right) = \left(\frac{r}{r_i}\right)^{\frac{(1+k^2)\tan \alpha}{k-k^2\tan \alpha}} \tag{A8}$$

Finally, equation (A8) is substituted into equation (A7) to obtain

$$\lim_{\substack{\phi \to \pi \\ \phi_i \to \pi}} \left(\frac{h}{R}\right) = \left(\frac{r}{r_i}\right)^{\frac{k+\tan \alpha}{k-k^2\tan \alpha}} \frac{(1 - k \tan \alpha)\cos \alpha}{(1 + k^2)^{1/2}} \tag{A9}$$

Therefore, equations (A6) and (A9) give the limiting values of $\frac{h}{R}$ at $\phi = 0$ and $\phi = \pi$ radians, respectively. It should be noted that equation (A9) gives the same relation as equation (A6) when α is replaced with $-\alpha$.

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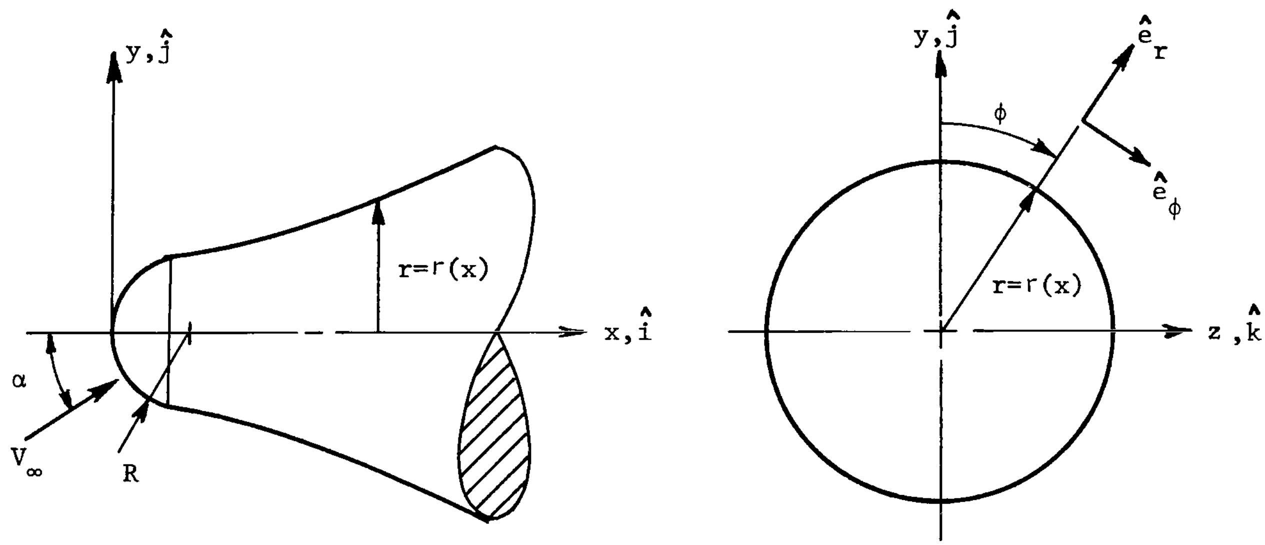

Figure 1.- Typical axisymmetric body illustrating axes and symbols used.

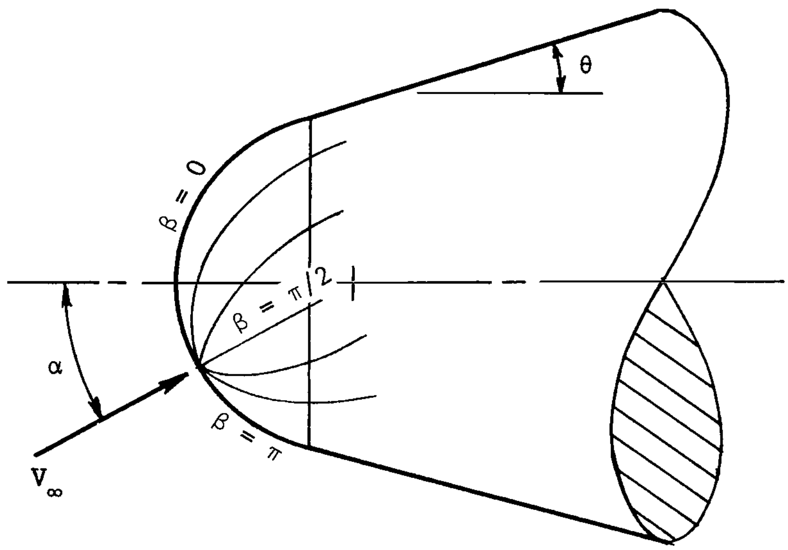

Figure 2.- Sketch illustrating streamline coordinate β.

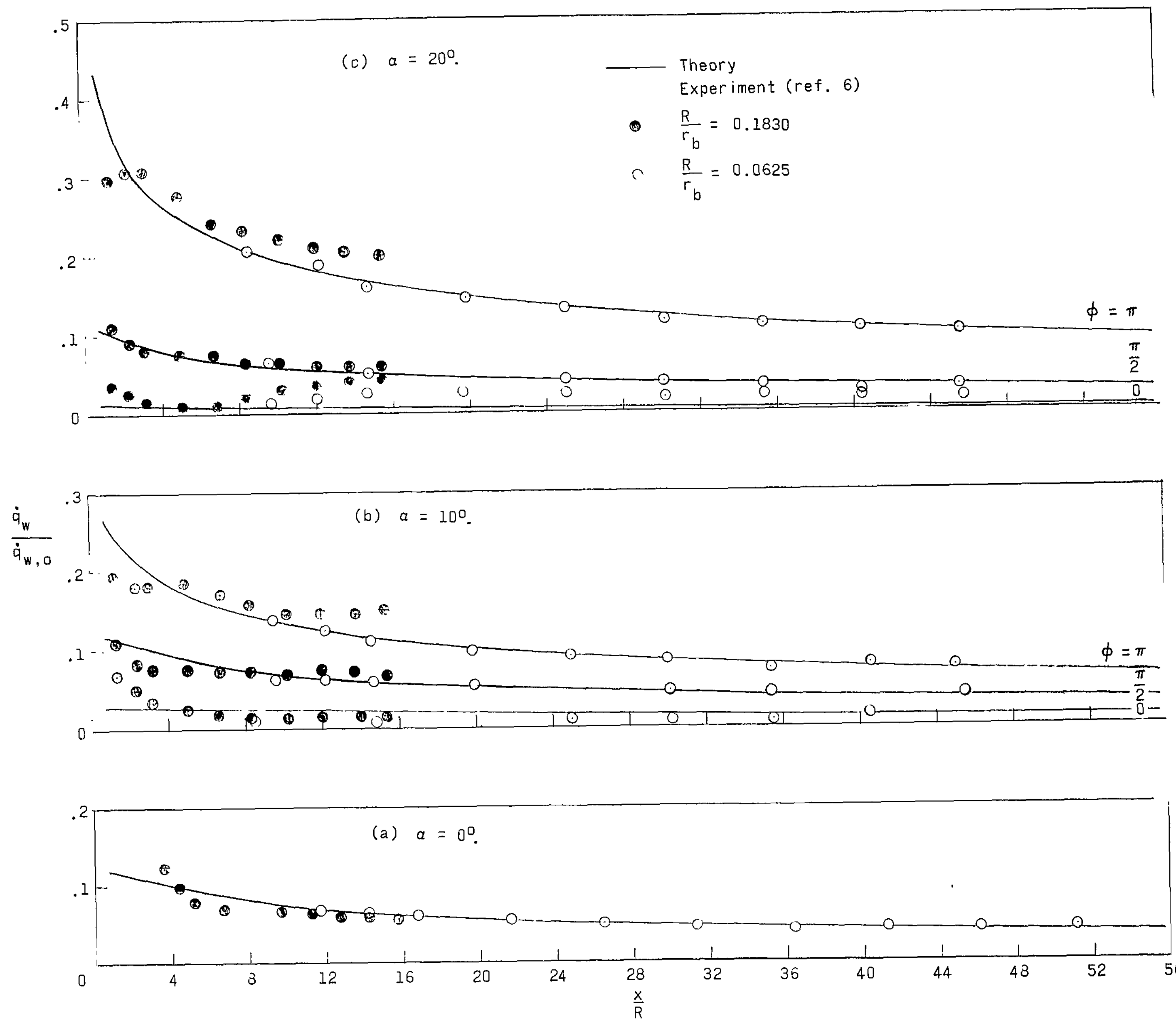

Figure 3.- Longitudinal variation of nondimensional heating-rate ratio $\dot{q}_w/\dot{q}_{w,o}$ on a spherically blunted cone with 15° semiapex angle. $M_\infty = 10.6$.

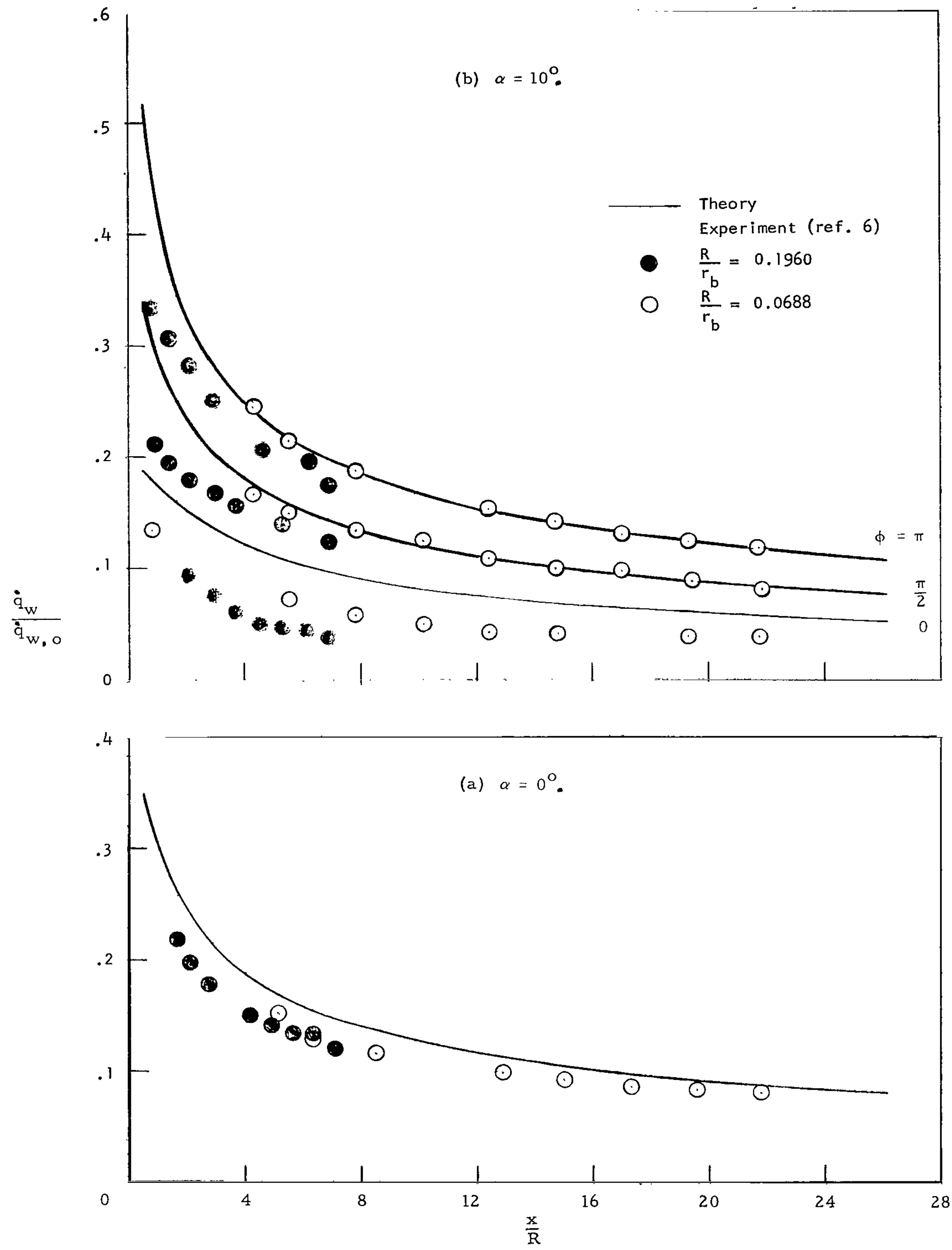

Figure 4.- Longitudinal variation of nondimensional heating-rate ratio $\dot{q}_w/\dot{q}_{w,0}$ on a spherically blunted cone with 30° semiapex angle. $M_{\infty} = 10.6$.

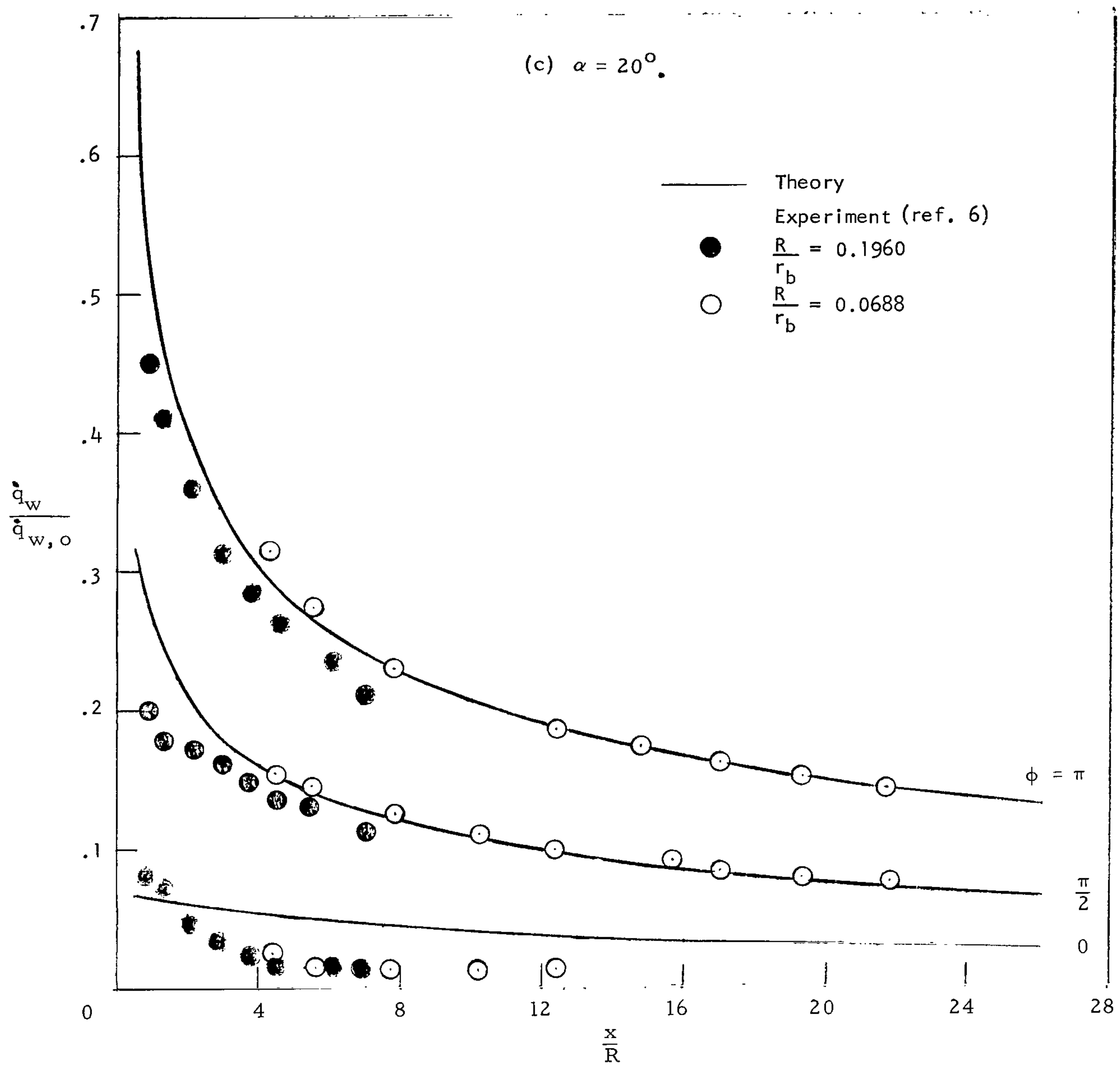

Figure 4.- Concluded.

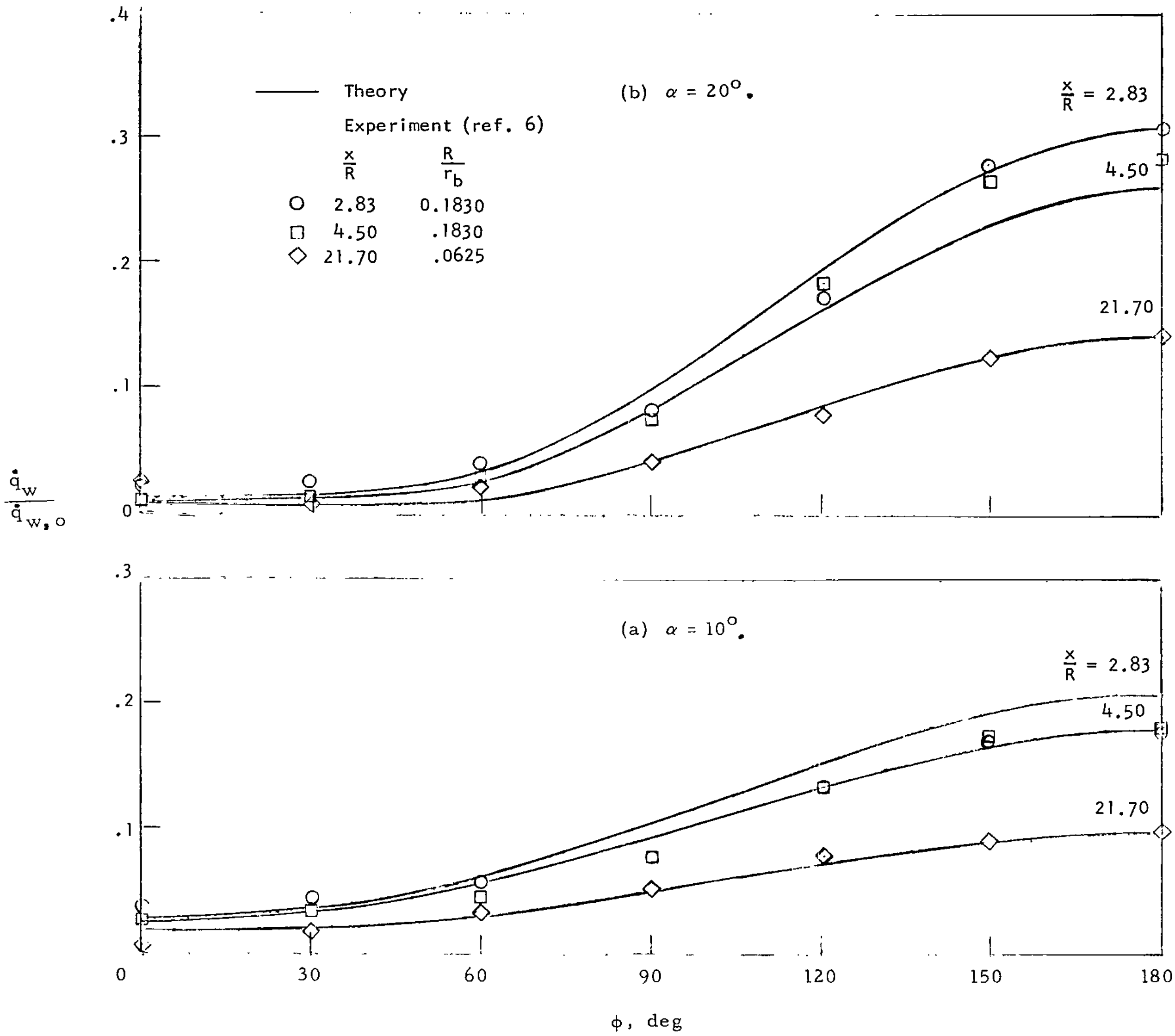

Figure 5.- Circumferential variation of nondimensional heating-rate ratio $\dot{q}_w/\dot{q}_{w,o}$ on a spherically blunted cone with 15° semiapex angle. $M_\infty = 10.6$.

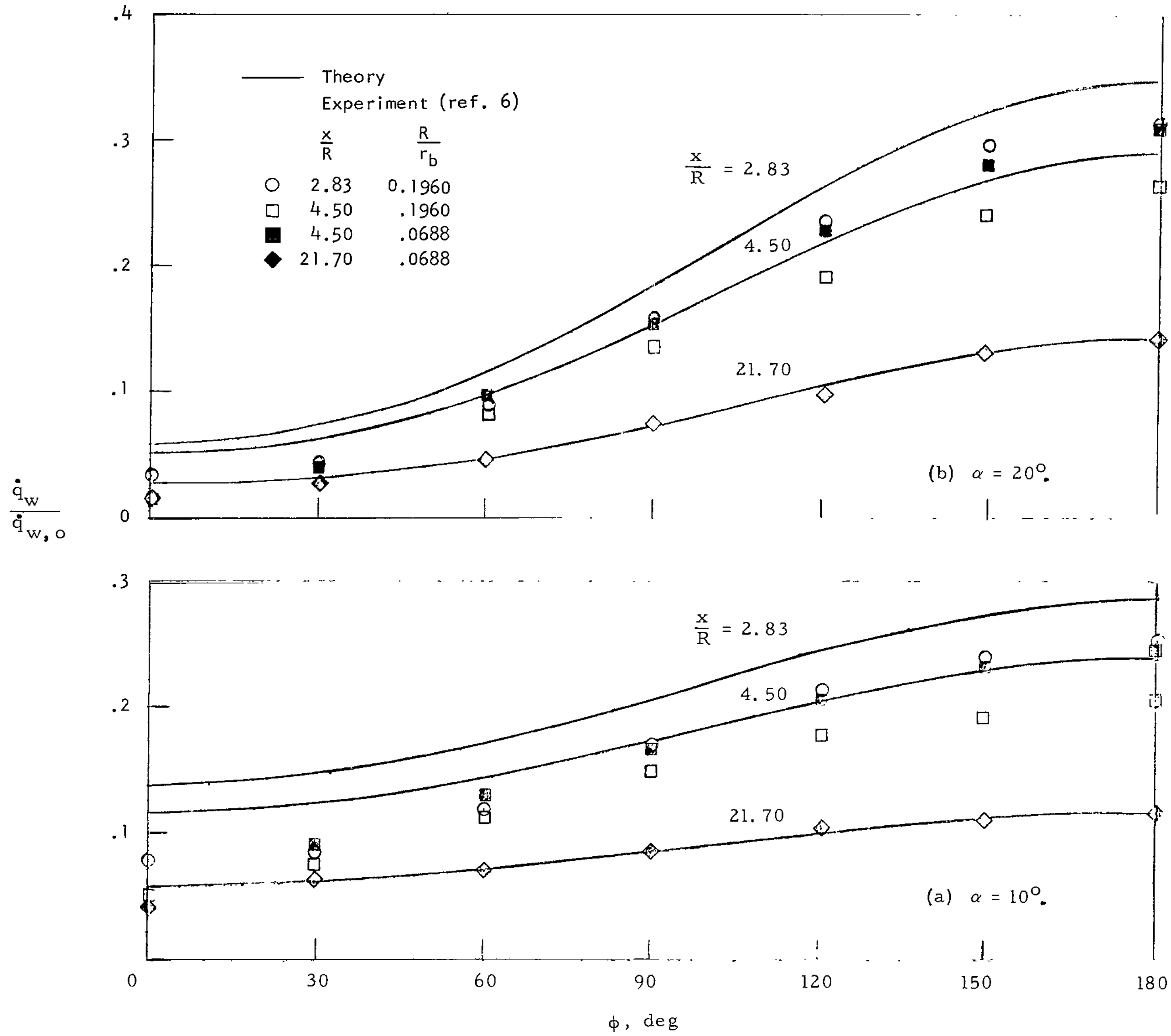

Figure 6.- Circumferential variation of nondimensional heating-rate ratio $\dot{q}_w/\dot{q}_{w,o}$ on a spherically blunted cone with 30° semiapex angle. $M_\infty = 10.6$.

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